JOURNAL OF AIRCRAFT
Vol. 34, No. 1, January–February 1997

Engineering Notes

ENGINEERING NOTES are short manuscripts describing new developments or important results of a preliminary nature. These Notes cannot exceed 6 manuscript pages and 3 figures; a page of text may be substituted for a figure and vice versa. After informal review by the editors, they may be published within a few months of the date of receipt. Style requirements are the same as for regular contributions (see inside back cover).

Indicial Aerodynamics in Compressible Flow–Direct Computational Fluid Dynamic Calculations

Vasudev Parameswaran* and James D. Baeder†
University of Maryland,
College Park, Maryland 20742

Introduction

THE accurate simulation of the aerodynamic loads on a helicopter, for example, requires the proper modeling of unsteady aerodynamics. Unsteady aerodynamics is particularly important in the prediction of the aeroelastic effects, control effectiveness, and flight performance because of the associated amplitude overshoots/undershoots and the aerodynamic phase lags. Indicial theory provides a method by which the response (aerodynamic loads), because of an arbitrary variation in the input (pitching, flapping, gust, etc.), can be calculated via the Duhamel integral if the indicial response is known (i.e., assuming linearity of the response with respect to the input). The indicial response of a quantity (e.g., lift), with respect to any of its influencing parameters (e.g., angle of attack α or pitch rate α), is the response to a step input of the influencing parameter. Indicial response is a very important starting point for a general time domain aerodynamic theory because it enables the calculation of the aerodynamic loads at a relatively low computational cost, compared to an actual on-line calculation. At the same time, the method retains the nominal accuracy of the flow predictions.

The indicial response is a mathematical concept and cannot be determined directly by experiment. However, it is possible to relate back to the indicial response from the frequency domain.[1] There exists exact closed-form analytical solutions for the indicial responses in inviscid incompressible flow.[2] However, there are no exact closed-form analytical solutions for compressible flow for all time. The initial value of the response (viz., noncirculatory loads) can be arrived at by use of linear piston theory, whereas the final value is given by a steady-state method. The variation of the response for intermediate time has been the topic of much research. Lomax[3] obtained exact analytical results for the indicial responses caused by a step change in angle of attack, a step change in pitch rate, and for the penetration of a sharp-edged gust in subsonic compressible flow. However, the results are only known for a short period of time (less than one chord of travel). Leishman[1] used a semi-empirical approach for the indicial responses, which are obtained from frequency domain measurements. Computational fluid dynamic (CFD) calculations are the next best choice for use in determining the indicial responses. The results from CFD test cases need to be validated with reliable experimental or exact theoretical results. Then, CFD can be used to considerable advantage for generating important data that are not currently available experimentally or to correct data that could be spurious because of scatter. Another advantage of using CFD is to obtain the time history of the pressure distributions around the airfoil, and also to observe physical features in the flow such as wave propagation and vorticity convection.

The incorporation of a step change into CFD methods, necessary for direct indicial response calculations, has proven to be quite challenging. Therefore, previous approaches in using Euler/Navier–Stokes methods to determine the indicial responses have been indirect,[4] wherein a known input is applied that varies smoothly with time (e.g., smoothened ramp function), so that the flow is not exposed to any discontinuities. The Laplace transform approach is then used to extract the indicial response by calculating and making use of the transfer function in the frequency domain. While this approach has been used, and has been useful, only the integrated loads can be determined. It may not be possible to derive insight into the features of the flow, or to obtain the pressure distributions around the airfoil, which may be desired. Thus, this approach does not exploit the full power and usefulness of CFD approaches.

There are certain problems associated with using Euler/Navier–Stokes methods to determine the indicial responses directly if the step input is applied on the airfoil (as a boundary condition). This results in numerical oscillations, and produces nonphysical features in the flow because of the infinite time derivative (the derivative becomes very large since the time-step size cannot be made zero). This may also result in numerical instabilities. However, this approach has been successfully applied to transonic small disturbance methods that utilize small disturbance boundary conditions.[5] However, with this approach, the step input in one of the parameters may change another input, and the output obtained may not be the correct indicial response because of the applied input. For example, if the airfoil is suddenly exposed to the flow at a different angle of attack, then the airfoil will also experience an infinite pitch rate (or a very large quantity). Then, the response (e.g., lift) will certainly be a reflection of a step change in angle of attack as well as an impulsive change in the pitch rate. Thus, decoupling between the parameters is practically impossible. This is more readily apparent in determining the indicial response caused by the pitch rate term.

In the present work, an alternate method has been implemented into a Euler/Navier–Stokes solver to directly calculate the indicial responses of an airfoil with respect to step changes in angle of attack and pitch rate, as well as the penetration of a sharp-edged gust. The formulation of the method leads to a natural decoupling of the parameters that influence the airfoil loading, while at the same time, circumventing the difficulties associated with incorporating step changes in the input at the airfoil boundary.

Approach

Unsteady solvers are written to incorporate so-called grid velocity, which is used to model unsteady flow via grid move-

Received Aug. 5, 1996; revision received Oct. 1, 1996; accepted for publication Oct. 14, 1996.

*Research Assistant, Center for Rotorcraft Education and Research, Department of Aerospace Engineering. Student Member AIAA.

†Assistant Professor, Center for Rotorcraft Education and Research, Department of Aerospace Engineering. Senior Member AIAA.

ment. Physically, grid velocity can be thought of as the velocity of a grid point during the unsteady motion of the airfoil. The idea is to incorporate the step change in input as a step change in grid velocity over the entire flow domain instead of only at the airfoil surface. This equivalence is, in some sense, similar to the equivalence between the flow over a moving airfoil in a stationary fluid, and the flow over a stationary airfoil in moving air. For example, a step change in the angle of attack is incorporated as a step change in vertical velocity all over the flow domain. This is indeed equivalent to a step change in the angle of attack, because the airfoil sees the flow as a step change in angle of attack. Also, there is no influence of the pitch rate term because the airfoil is not made to pitch, and because the step change is enforced throughout the flow domain and uniformly, the airfoil is exposed only to a pure angle of attack. This results in a decoupling of the angle-of-attack time history and the pitch rate time history. Similarly, to obtain the indicial response caused by the pitch rate term, the entire flow domain is given a rotation, by imposing a grid velocity that varies linearly with the distance from the rotation point (typically the quarter-chord of the airfoil). This corresponds to $v(x, y) = \alpha r(x, y)$, where $v(x, y)$ denotes the grid velocity and $r(x, y)$ denotes the distance of the grid point located at (x, y) from the point of rotation. Once again, the airfoil is exposed to pure pitching and not to any direct change in the angle of attack. Here again, the effects of the angle-of-attack history and the pitch rate history are properly decoupled. In addition, this approach does not produce any numerical oscillations, but does produce results that agree exceedingly well with the exact solutions available in the initial stages.

Results

The computations were performed using a modified two-dimensional version of the TURNS code,[6] on three different grid resolutions, namely a coarse 101 × 21 mesh, a moderately fine 181 × 31 mesh, and a fine 251 × 61 mesh to ascertain grid size independence of the calculations. The resolution of the grid was found to have little or no effect on the time histories of the lift. Similarly, to ascertain time-step independence, the calculations were performed for time-step sizes of $d\hat{t}$ = 0.01 and 0.05 on the 181 × 31 mesh (where $\hat{t} = ta_\infty/c$, where a_∞ is the freestream speed of sound and c is the airfoil chord). The time-step size change was found to have negligible influence on the lift time histories. Specific computations were performed for a NACA 0006 airfoil at freestream Mach numbers of M_∞ = 0.3, 0.5, 0.65, and 0.8 on the 181 × 31 mesh with a time step of $\hat{t}$ = 0.05 for a step change in angle of attack of α = 0.08 rad and a step change in pitch rate of 0.0035 rad/s. The gust response calculations were performed on the same airfoil with a gust velocity equal to 0.08 times the freestream velocity, such that it induced a net change in angle of attack of attack of approximately 4 deg in each case.

Figure 1 shows the computed coefficients of lift, for a step change in angle of attack, normalized by the angle of attack imposed, as a function of $s = 2V_\infty/c$, the nondimensional aerodynamic time. The results have been plotted for M_∞ = 0.3, 0.5, 0.65, and 0.8. One important observation to make is that the computed values for the first few time steps may be incorrect. This is because of the transients involved in the sudden change in the grid velocity. However, the computed values after the first few time steps are expected to be correct since the transients have decayed to negligible values. The initial lift values (taking the initial values to be the maximum reached when s is close to zero) match the exact linear piston-theory value of $4/M_\infty$ very closely, while the final values are close to the linearized steady-state value of $2\pi/\beta$, where $\beta = \sqrt{1 - M_\infty^2}$. The indicial response is a combination of noncirculatory (wave propagation) and circulatory effects. It is known that for incompressible flow the noncirculatory force at $s = 0$ is infinite (Dirac delta function), whereas the circulatory force is half of the final value. For $s > 0$, the noncirculatory component is 0, while the circulatory force builds and reaches its maximum value of 2π at $s = \infty$. For compressible flow, however, the speed of sound being finite produces two effects. The noncirculatory component is finite at $s = 0$ and is nonzero for $s > 0$. However, it decays rapidly. Lomax[3] derived exact closed-form expressions of the indicial responses for small times for a flat plate airfoil in linearized compressible flow. The expression is given by

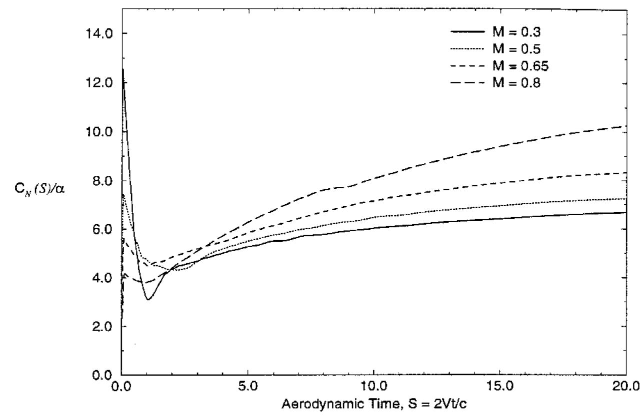

Fig. 1 Indicial response for step change in α using CFD.

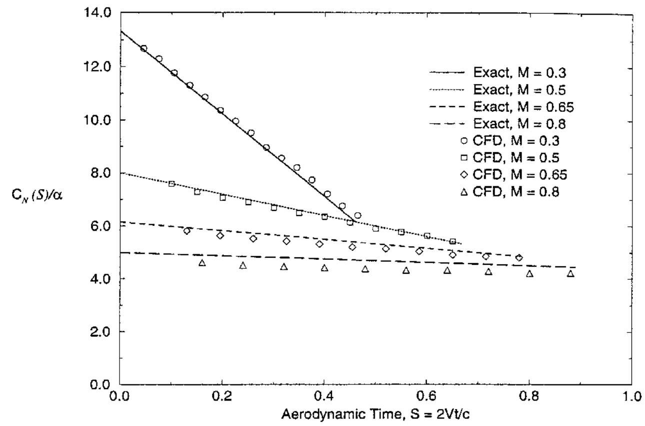

Fig. 2 Comparison with exact results for step change in α for small times.

$$C_N(s)/\alpha = (4/M_\infty)\{1 - [(1 - M_\infty)/2M_\infty]s\}$$
$$0 \leq s \leq 2M_\infty/(1 + M_\infty) \qquad (1)$$

Figure 2 shows a comparison of the computed and exact results at small times. Again, bearing in mind the error resulting from the transients during the first few time steps, the data points corresponding to the first two time steps have been excluded from the plot. It can be seen that the agreement is excellent. It should also be noted that since the exact solutions are based on linear theory, the agreement is expected to be progressively worse for higher Mach numbers, especially when the flow becomes supercritical. As expected, the comparison is excellent for M_∞ = 0.3 and 0.5, whereas the agreement reduces progressively through M_∞ = 0.8 as nonlinear effects become more important.

Figure 3 shows the computed results for the gust responses, again for small times. Exact analytical results are known for this case,[3] and are shown in the figure for a comparison. The expression is given by

$$C_N(s)/(w_g/V_\infty) = 2s/\sqrt{M_\infty}, \qquad 0 \leq s \leq 2M_\infty/(1 + M_\infty) \qquad (2)$$

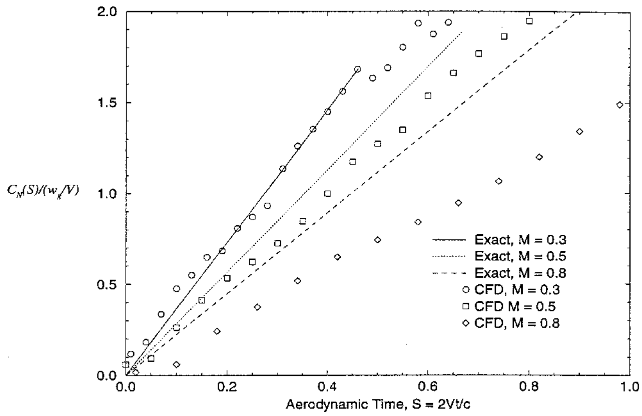

Fig. 3 Comparison of CFD with exact results for gust response at small times.

where w_g is the gust vertical velocity. The comparison is excellent for $M_\infty = 0.3$, whereas the agreement becomes progressively worse for $M_\infty = 0.8$.

Recently, this same method has been extended to be three-dimensional to examine the indicial response of wings to a step change in angle of attack and the resulting detailed flowfield development.[7]

Conclusions

A method has been developed to calculate the indicial and gust responses of an airfoil in compressible flow directly using CFD. Previous work with Euler/Navier–Stokes codes have been restricted in the sense that the indicial responses can be calculated only indirectly using the Laplace transform approach. Furthermore, such methods cannot give the surface pressure time histories, and hence, may not be useful in gaining insight into the physical features of the flow. The present method, using a grid-velocity approach, has been demonstrated to be accurate via comparison with known exact analytical results at $t = 0$ and $t = \infty$ and via comparison with exact linear theory results for small times. The comparison is excellent for $M_\infty = 0.3$, whereas the agreement becomes progressively worse as the flowfield becomes increasingly nonlinear.

Acknowledgments

This work was partially supported by the U.S. Army Research Office under the Smart Structures University Research Initiative, Contract DAAL03-92-G-0121. Gary Anderson served as the Contract Monitor. Additional support was provided by the Minta Martin Fund for Aeronautical Research, from the University of Maryland, College Park, Maryland.

References

[1]Leishman, J. G., "Indicial Lift Approximations for Two-Dimensional Subsonic Flow as Obtained from Oscillatory Measurements," *Journal of Aircraft*, Vol. 30, No. 3, 1993, pp. 340–351.

[2]Bisplinghoff, R. L., Ashley, H., and Halfman, R. L., *Aeroelasticity*, Addison–Wesley, Reading, MA, 1955.

[3]Lomax, H., "Indicial Aerodynamics," *AGARD Manual of Aeroelasticity*, Pt. II, Nov. 1960, Chap. 6.

[4]Lesieutre, D. J., Reisenthel, P. H., and Dillenius, M. F. E., "A Practical Approach for Calculating Aerodynamic Indicial Functions with a Navier-Stokes Solver," AIAA Paper 94-0059, Jan. 1994.

[5]Ballhaus, W. F., and Goorjian, P. M., "Computation of Unsteady Transonic Flows by the Indicial Method," *AIAA Journal*, Vol. 16, No. 2, 1978, pp. 117–124.

[6]Srinivasan, G. R., Baeder, J. D., Obayashi, S., and McCroskey, W. J., "Flowfield of a Lifting Rotor in Hover: A Navier-Stokes Simulation," *AIAA Journal*, Vol. 30, No. 10, 1992, pp. 2371–2378.

[7]Singh, R., and Baeder, J. D., "The Direct Calculation of Indicial Lift Response of a Wing Using Computational Fluid Dynamics," AIAA Paper 95-2508, June 1996.

Flutter Analysis Using Unsteady Aerodynamics in Non-Pade Form

E. Nissim*
Technion—Israel Institute of Technology, Haifa 32000, Israel

Introduction

THE linearized equations of motion (EOM) of a flexible aircraft contain unsteady aerodynamic terms that depend on the Mach number M and the reduced frequency k. The exact dependence of the aerodynamic coefficients on M and k cannot be expressed in the form of algebraic functions. As a result, these coefficients are computed for each desired Mach number for a set of predetemined values of reduced frequencies. These values will be referred to as the tabular values of the complex aerodynamic coefficients. The classical methods of solution[1–3] of the flutter equations involve the k, p, and $p-k$ methods (with the Pade method[4–6] considered as a form of the p method). The k method leads to complex EOM, which are solved using the tabular values of the aerodynamic coefficients obtained for simple harmonic oscillations. Therefore, the complex eigenvalues obtained before and after the flutter point do not reflect the damping of the different structural modes. The p and the $p-k$ methods yield complex eigenvalues that attempt to represent the actual damping of the structural modes. The $p-k$ method yields EOM that may assume either complex coefficients[1] or real coefficients,[3] and it is based on the tabular values of the aerodynamic coefficients (or on interpolated values, for values of k between adjacent tabular points), without attempting to fit them into any explicit functional form. The EOM in the $p-k$ method are solved in an iterative fashion so that the assumed value of k (for which oscillatory aerodynamic coefficients are either available as tabular values, or as interpolated values), converges to the computed value of the imaginary part of a preselected eigenvalue (at a chosen airspeed). The iterations are repeated, for a single mode at a time, until all of the modes are treated for convergence in the manner just described. The p method avoids the previous iteration process by using explicit expressions that approximate the transient aerodynamics in the time domain. The main difficulty with the p method lies in the derivation of the approximating expressions in the time domain for configurations that employ lifting surface aerodynamics. The Pade method, which is essentially a p method, is based on using the tabular values of the complex aerodynamic coefficients, derived for oscillatory motion, for the purpose of fitting them into an explicit expression involving rational polynomials that are eventually used in the Laplace domain. The most common Pade representation[4] assumes the following form:

$$A(k) = A_0 + A_1 ik + A_2(ik)^2 + \sum_{j=1}^{n_L} \frac{A_{2+j} ik}{(ik + \beta_j)} \qquad (1)$$

where all of the A_j are real matrices, β_j is a real lag term, and $i = \sqrt{-1}$. The Pade representation enables one to cast the aeroelastic EOM into a set of first-order differential equations with constant coefficients, a form that enables the use of optimal control theory for the design of control laws to suppress flutter and/or improve the aeroelastic behavior of the structure. However, the introduction of the rational terms, referred to also

Received Oct. 23, 1995; revision received Aug. 30, 1996; accepted for publication Sept. 20, 1996.

*Professor, Faculty of Aerospace Engineering. Associate Fellow AIAA.